DISCUSSIONS.

RELATING TO INDETERMINATE FORMS.

I. A QUESTION WITH RESPECT TO 0/0.

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Has 0/0, (*a*) in particular cases a definite value, or (*b*) is it always indeterminate?

A generation ago mathematicians, in general, answered (*a*) affirmatively and (*b*) negatively; but in recent years, as a rule, they do just the reverse. This change is due to the ruling out of division by zero, because, as is asserted, it is impossible in cases like $a/0$ and indeterminate in cases like 0/0.

Without advocating the one contention above the other, I invite a careful consideration of the following argument, hoping that some one will point out the fallacy, if fallacy there be.

The locus of

$$y = \frac{x^2 - a^2}{x - a} \tag{1}$$

consists of the two straight lines[1]

$$y - x - a = 0 \tag{2}$$

and

$$x - a = 0, \tag{3}$$

as shown by clearing of fractions, transposing and factoring.

Now when $x = a$ in (1), *y has a definite and also an indeterminate value.*

For, according to the definitions of loci and their equations, the value of y when $x = a$ is the ordinate of the point of intersection of the loci of $x - a = 0$ and (1); that is, of the line $x - a = 0 \cdots$ (4) and the two lines (2) and (3).

But the point of intersection of (4) and (2) is $(a, 2a)$, and that of (4) and (3) is $(a, 0/0)$.

Therefore, when $x = a$ in (1) we have

$$y = 2a \quad \text{and} \quad y = 0/0.$$

It will be seen that the above process of determining the definite value $2a$ is the simple operation of finding the intersection of two given lines, and is independent of the idea or doctrine of limits.

By the same method the following more general proposition may be proved: *When $x = x'$ the definite value of*

$$\frac{f(x) - f(x')}{x - x'} \quad \text{is} \quad f_1(x),$$

in which $f_1(x)$ is the derivative of $f(x)$.

[1] See, for instance, J. W. Young's *Fundamental Concepts of Algebra and Geometry*, p. 214; Granville's *Differential and Integral Calculus*, p. 171; Townsend and Goodenough's *Essentials of Calculus*, p. 21; McMahon and Snyder's *Differential Calculus*, p. 116; and many others.